Spoon or slide?

The non-linear matter power spectrum in the presence of massive neutrinos

Hannestad, Upadhye, and Wong,
JCAP 11 (2020) 062, arXiv:2006.04995

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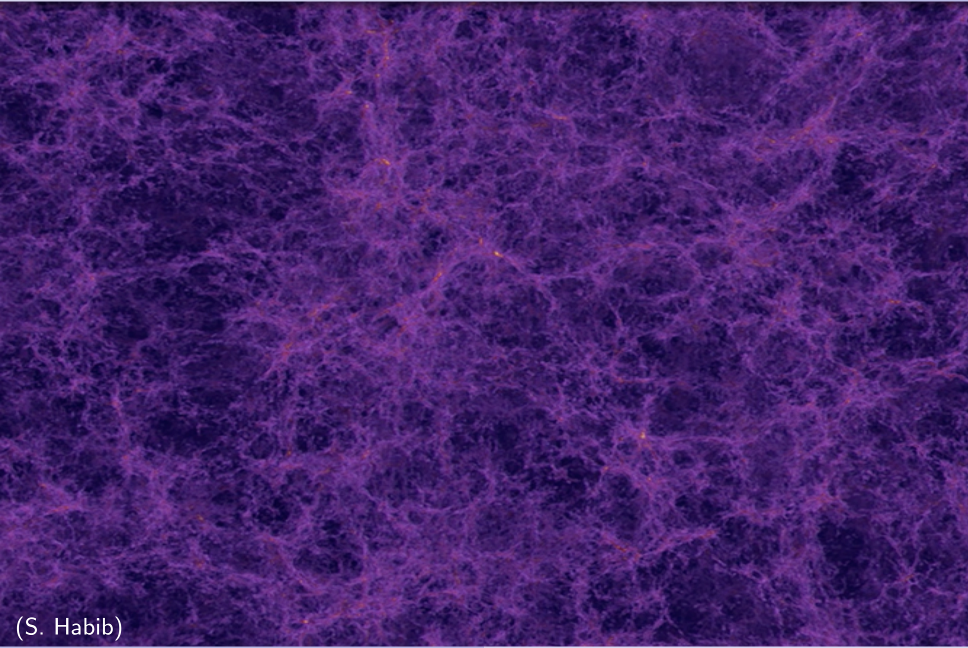
Journal Club
November 8, 2023

Large-scale cosmological structure



(S. Habib)

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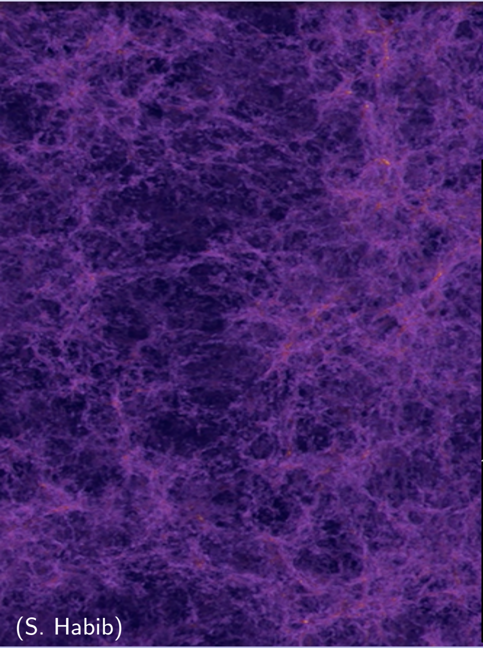

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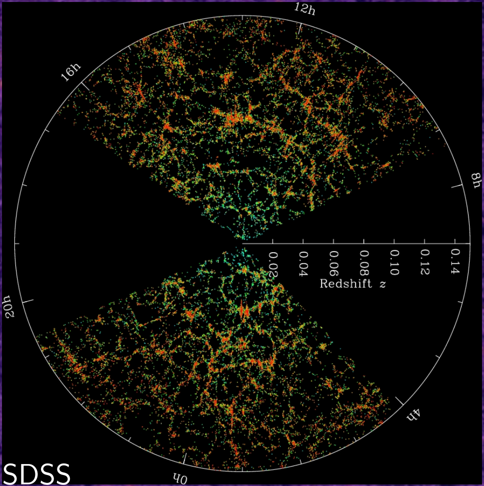

SDSS

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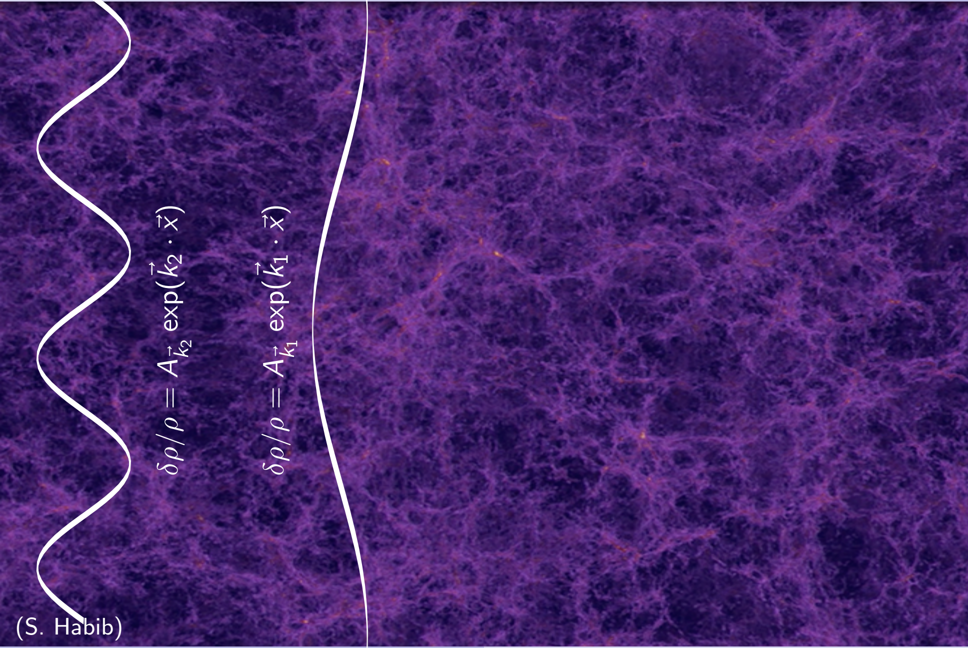

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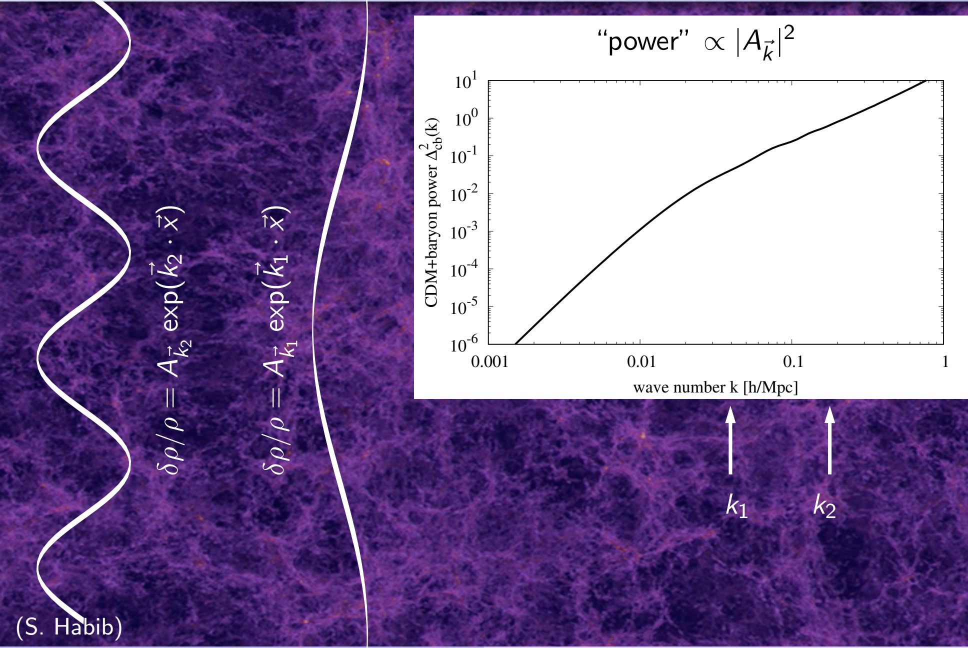

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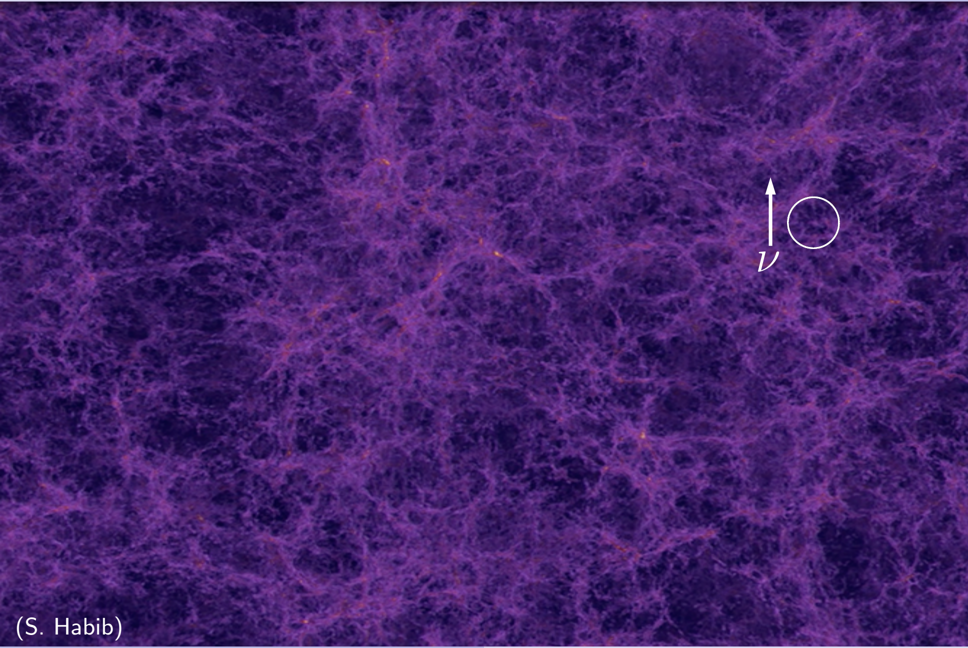

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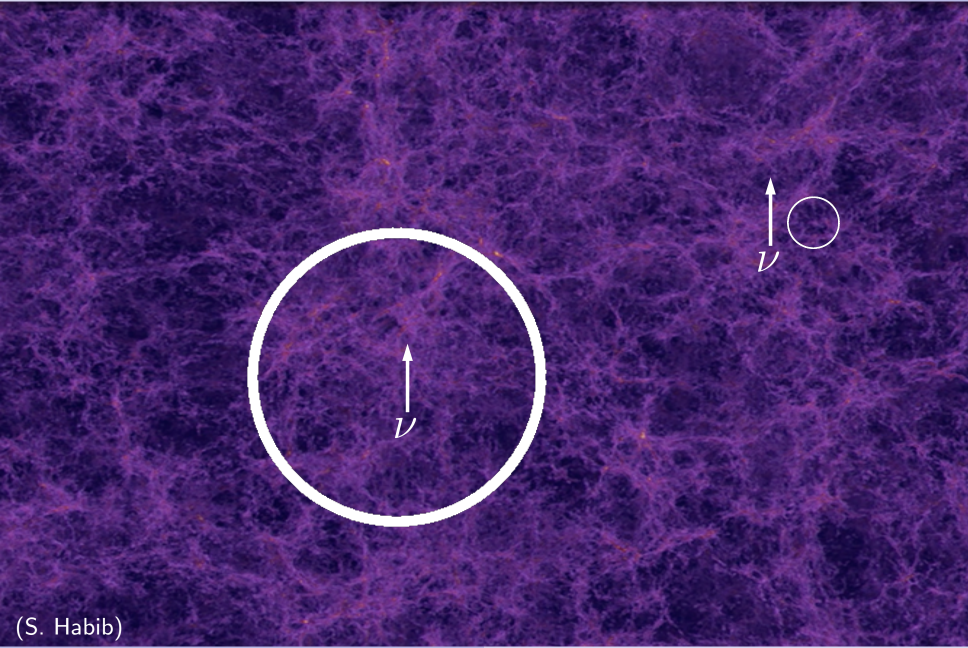
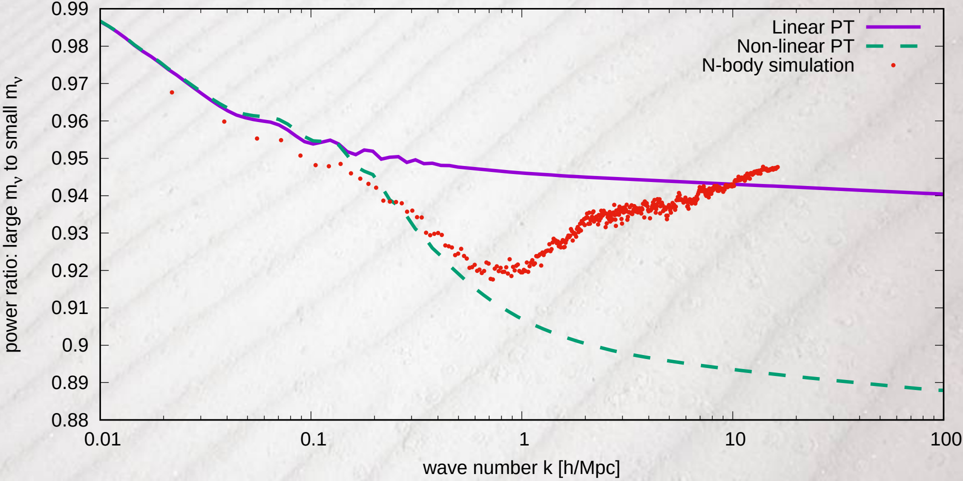


Neutrino suppression: spoon or slide?



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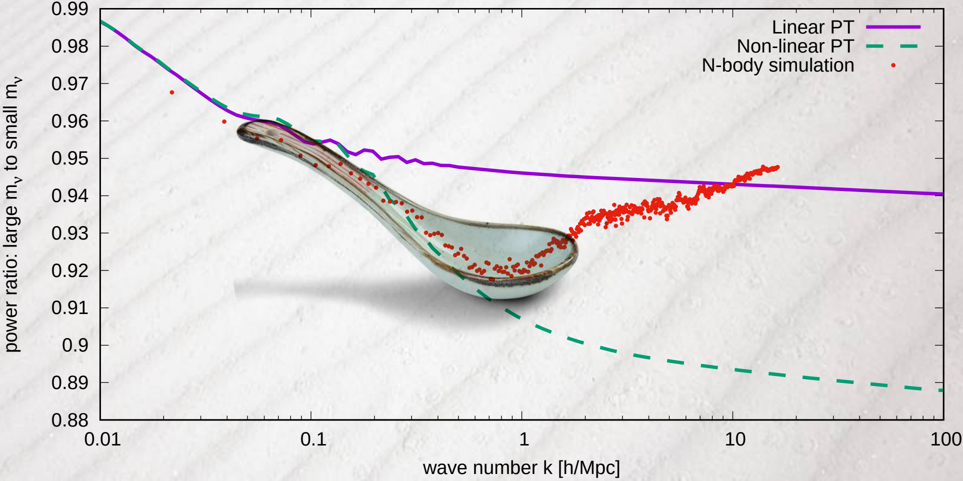

Halo model: beyond perturbation theory

The halo model of large-scale structure assumes that all Cold Dark Matter (CDM) and baryons is in collapsed objects called halos.

It has three ingredients:

1. spherical collapse approximates halo size and density contrast;
2. the mass function counts halos of each mass; and
3. the density profile provides the internal structure of halos.

Its power spectrum is the sum of the two-halo power (similar to perturbation theory) and the one-halo power: $\Delta^2(k) = \Delta^2_{\mathrm{2h}} + \Delta^2_{\mathrm{1h}}$.

Spherical collapse

Einstein-de Sitter universe [only CDM+baryons (CB)]
Linear spherical overdensity $\delta_0 = \rho_{\rm CB}/\bar{\rho}_{\rm CB} - 1$ today
$\Rightarrow$ $a\delta_0$ at scale factor a (fraction a of today's size)

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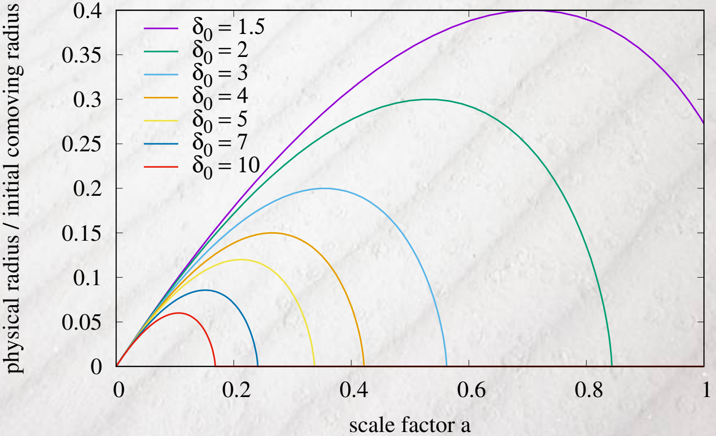

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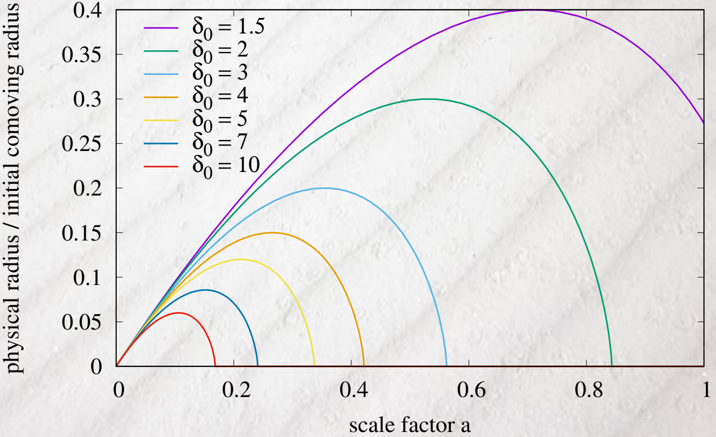

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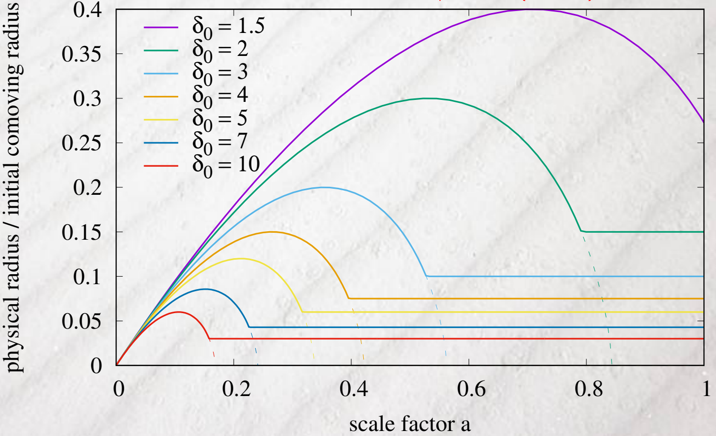

Mass function

overdensity right here: $\delta_{\vec{x}=\vec{0}} = \int \frac{d^3k}{(2\pi)^3}\delta(\vec{k})$

split integral so that $V_{\rm univ} \int_{k_i}^{k_{i+1}} \frac{d^3k}{(2\pi)^3} = 1$ $\Rightarrow \delta_{\vec{x}=\vec{0}} \approx \frac{1}{V}\sum_i \delta(\vec{k}_i)$

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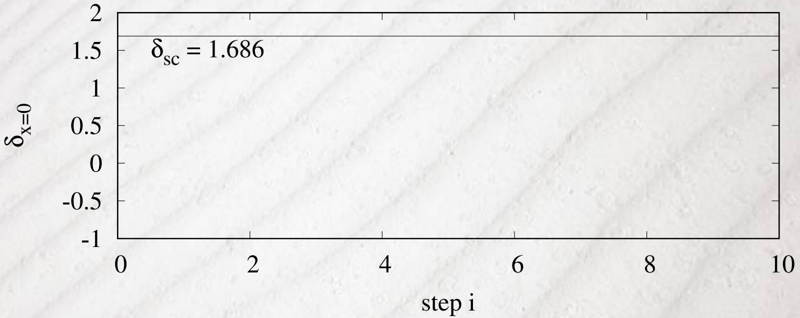

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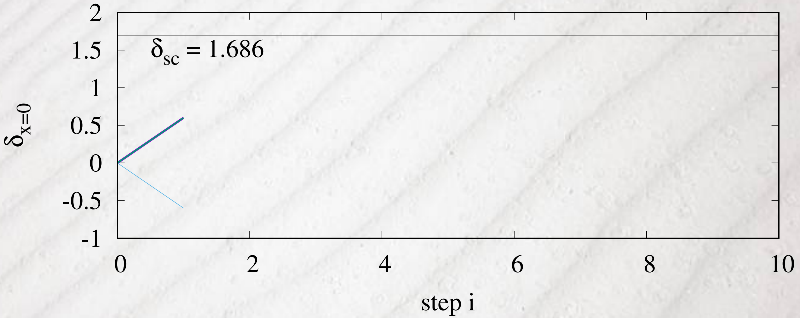

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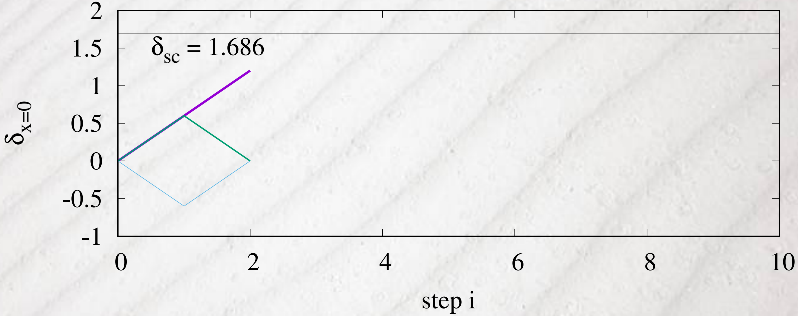

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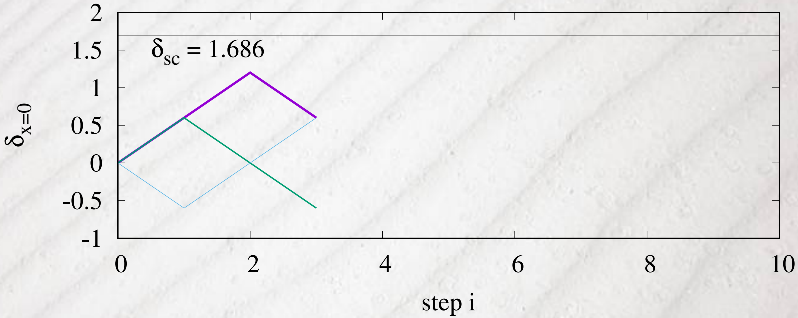

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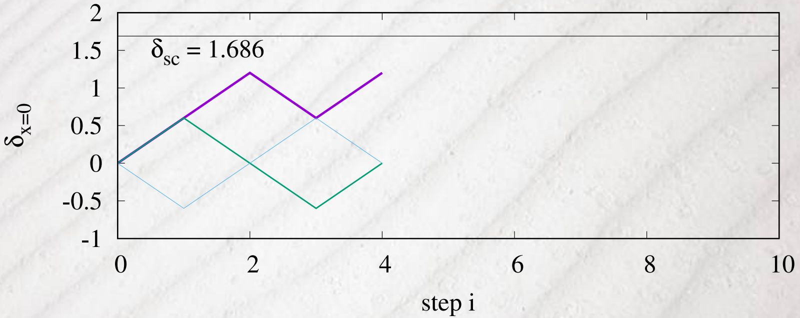

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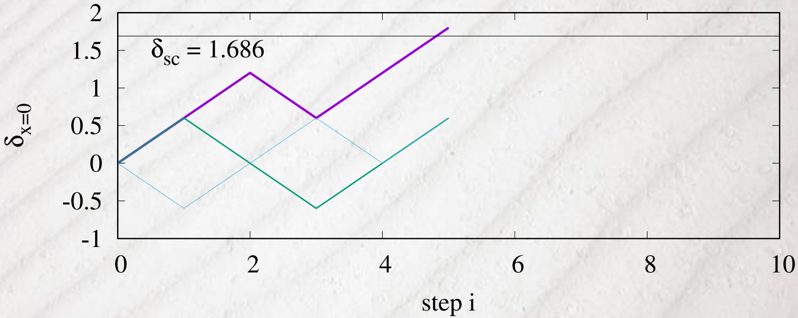

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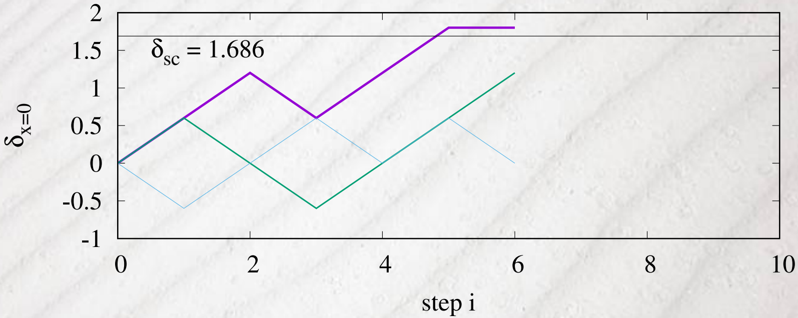

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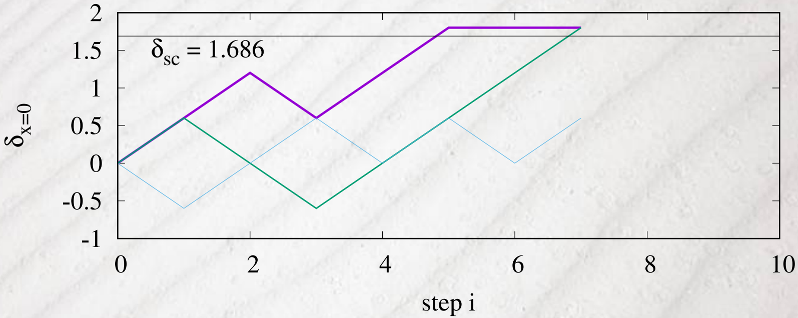

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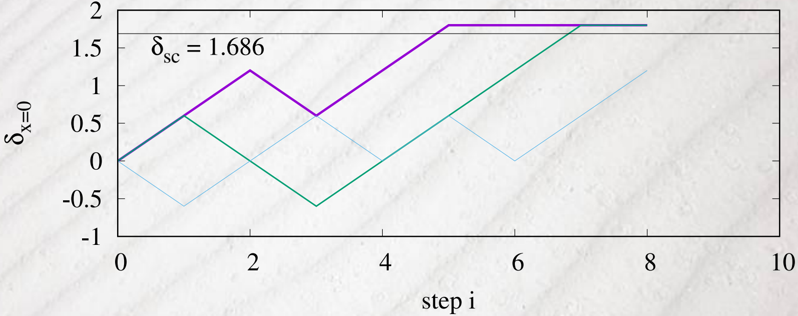

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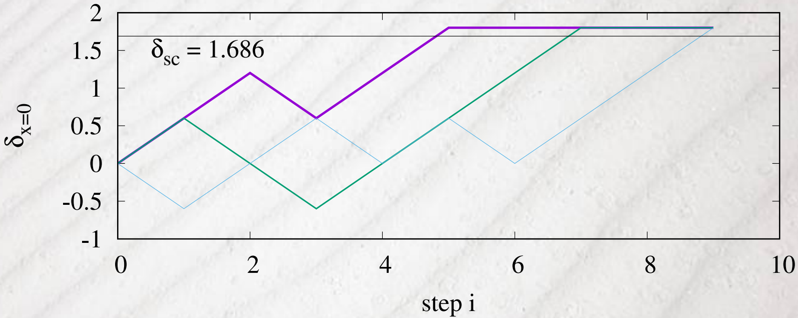

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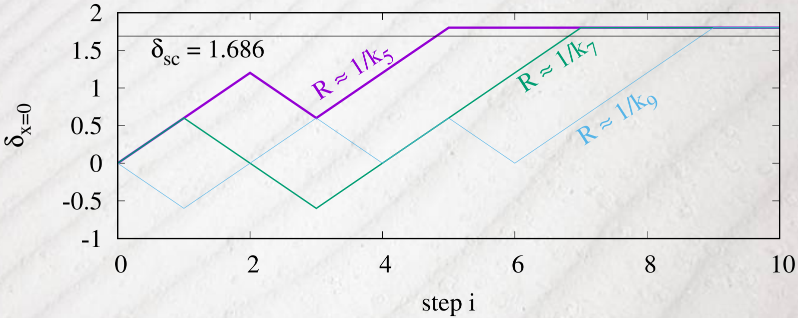

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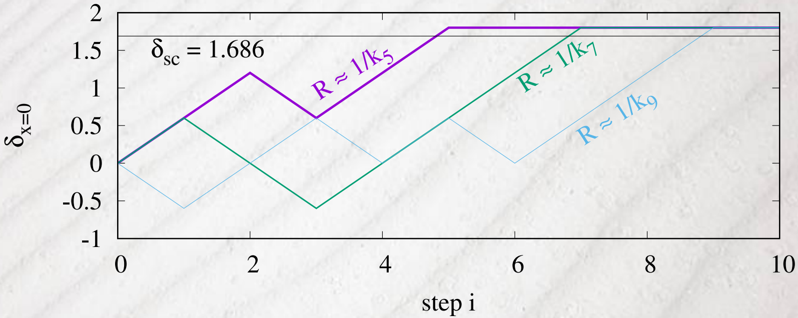

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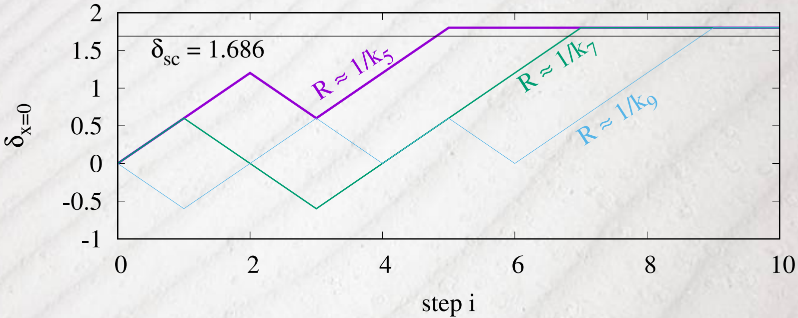

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halo rarity: $\chi(R) = \delta_{\rm sc}/\sigma(R) \Rightarrow R(\chi),\ M(\chi) = \frac{4}{3}\pi\bar{\rho}_{\rm CB}R(\chi)^3$

Mass function II

We can't predict the halo mass at a particular position, only the probability distribution, which we average to get a number density.

- number density with mass $\leq M$: $n(M)$
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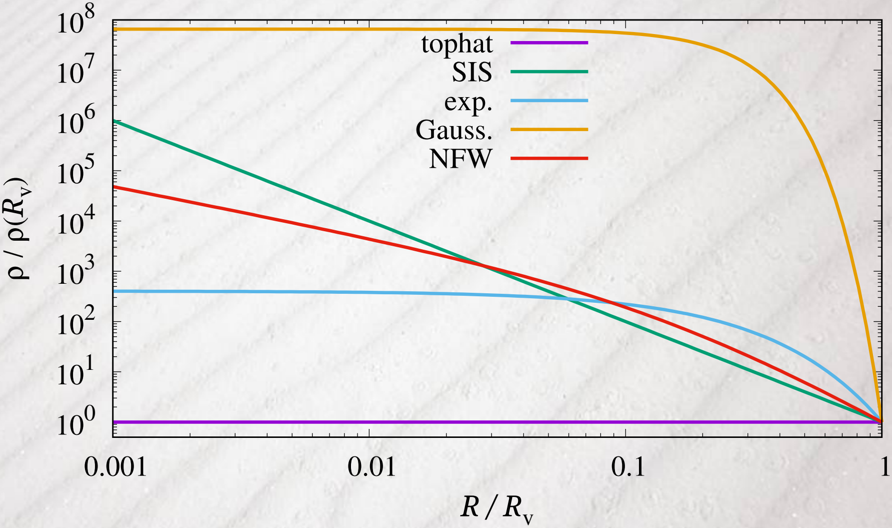
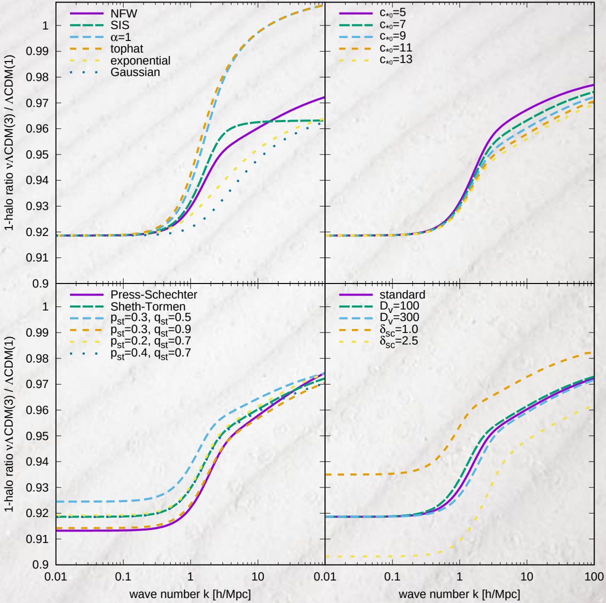
$f(\chi) = A\left(1 + \frac{1}{(q_{\mathrm{st}}\chi^2)^{p_{\mathrm{st}}}}\right)\exp(-q_{\mathrm{st}}\chi^2/2)$ with $p_{\mathrm{st}} = 0.3$, $q_{\mathrm{st}} = 0.707$

Sheth, Mo, Tormen (2001); Sheth, Tormen (2002)

Density profile



The one-halo power spectrum rises with k



General behavior of one-halo power spectrum

one-halo power: $\Delta^2_{\mathrm{1h}}(k) = \frac{2k^3}{3\pi}\int_0^\infty d\chi\, f(\chi) R(\chi)^3 U(kR(\chi)/\mathcal{V}_{\mathrm{v}})^2$

Fourier transform of spherically-symmetric density:
$U(s) = \frac{4\pi\mathcal{R}_{\mathrm{v}}^3}{M}\sqrt{\frac{\pi}{2s}}\int_0^\infty dy\, y^{3/2} J_{1/2}(sy)\rho(y\mathcal{R}_{\mathrm{v}})$

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  Example: SIS ($\rho(r) \propto r^{-2}$) $\Rightarrow U(kR/\mathcal{V}_{\rm v}) \to \pi/(2kR/\mathcal{V}_{\rm v})$
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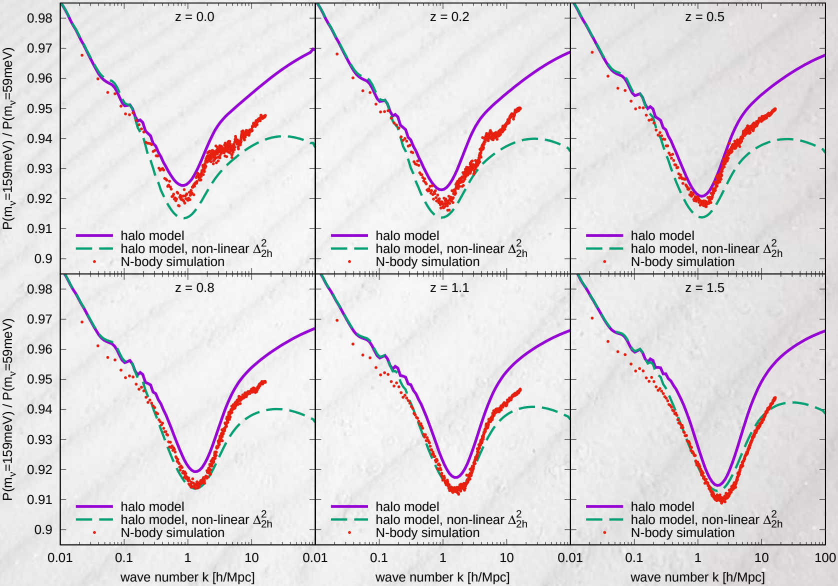
Halo centers are less cosmology-dependent than halo outskirts.

Spoon: Halo model vs. N-body simulation



Hannestad, Upadhye, and Wong, JCAP **11**:062 (2020), arXiv:2006.04995

Conclusions

1. The N-body neutrino spoon is a real, physical feature due to the transition from a two-halo suppression that increases with k to a one-halo suppression that decreases with k.
2. The decreasing one-halo suppression is a general prediction of the halo model, and arises from $U(s)$ switching from flat at low s to declining at high s.
3. With first-principles linear perturbation theory, a standard halo model from 2002 (6 years before the spoon was discovered) predicts the position and redshift-dependence of the spoon.
4. Using non-linear perturbation theory, this halo model also predicts the depth of the spoon for $z \gtrsim 1$.

Hannestad, Upadhye, and Wong, JCAP **11**:062 (2020), arXiv:2006.04995